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Mergers, investments and demand expansion[☆]

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HIGHLIGHTS

- A merger boosts investment in coverage for a new technology.
- When coverage is endogenous, a merger may raise total welfare and consumer surplus.
- Total coverage increases irrespective of whether coverage is observable or not before pricing.
- Total coverage increases when the new technology replaces a competitive old-generation.

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ABSTRACT

In this paper, we study the impact of a merger to monopoly on prices and investments. Two single-product firms compete in prices and coverage for a new technology. In equilibrium, one firm covers a larger territory than its competitor with the new technology, leading to single-product and multi-product zones, and sets a higher uniform price. If the firms merge, the merged entity can set different prices and coverage for the two products. We find that the merger raises prices and total coverage, but reduces the coverage of the multi-product zone. We also show that the merger can increase total welfare and consumer welfare.

1. Introduction

In a number of recent merger cases in Europe among mobile network operators, the potential impact of mergers on investment has been hotly debated.[1] Operators claim that mergers in the sector can foster the deployment of new technologies, while the European Commission has expressed the view that mergers are detrimental to investment absent efficiency gains.[2]

In this paper, we develop a simple model where a merger to monopoly raises investment in coverage of a new technology, despite the absence of synergies. As coverage is only one dimension at stake in a merger, our paper does not aim at providing a full analysis of mergers, but at delivering new insights that shed light on the operators' claim and should be factored in a merger case.

We consider a coverage-price game, where two firms decide on prices and coverage of a new technology over a territory.[3] When firms are separate, one firm covers a larger share of the

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1 See, for example, the European Commission decision on the Hutchison 3G/Orange merger case in Austria, COMP/M.6497 (December 2012).

2 Genakos et al. (2018) provide evidence that concentration in the mobile market may indeed imply a trade-off between prices and investments. Based on panel data for the period 2002–2014 covering 33 countries from Europe and the OECD, they find that a 4-to-3 merger raises prices by 16% on average, but at the same time increases investments by operator by 19%. However, the evidence of a positive effect of mergers on investment is not totally conclusive, since they find that total investment is not affected significantly by the merger.

3 In the telecommunication industry, roll-out of duplicated infrastructures occurs for mobile 4G and FTTH (for instance, in France and in Spain).

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territory than its rival. When a merger-to-monopoly takes place, the merged entity raises all prices, increases total coverage with a positive effect on welfare, and reduces the coverage of the multi-product zone, which can either harm welfare (due to lower variety) or increase it (due to the business-stealing effect). We provide an example where total welfare and consumer surplus can increase with the merger.

Our paper is related to Motta and Tarantino's (2017) finding that absent spillovers or synergies, the reduction of output by the merged entity induces a reduction of cost-reducing investment.[4] We identify a new effect that implies a positive impact of mergers on investment, when investment increases coverage. Other articles pointing to different channels which may lead to such a positive effect are Marshall and Parra (2017), in the context of a dynamic model of leadership, and Loertscher and Marx (2017), in a model with buyer power.[5] Our paper also builds on the literature on universal service in network industries, which focuses on regulatory issues (see, among others, Valletti et al., 2002; Hoernig, 2006; Gautier and Wauthy, 2010).

The model is presented in Section 2 and analyzed in Section 3. All the proofs are in Appendix.

2. Model

Consider a geographic market represented by a half-line from 0 to $\bar{z}$. Two operators, 1 and 2, deploy a new technology. Initially, the market is not covered at all and there is no alternative old-generation technology. The two operators have the same development cost $c(x)$ to deploy the technology in location x, where $c(x)$ is increasing. We define as

$$C(z) = \int_0^z c(x)\,dx$$

the total cost of covering the locations from 0 to z. We assume that $c(0)$ is small enough and $\lim_{x\to\bar{z}} c(x)$ is large enough so that in a duopoly both firms invest and no firm covers the whole market (see Footnote 7). We also assume that firm $i = 1, 2$ deploys the technology in all locations $x \leq z_i$ where $z_1 \geq z_2$.[6]

The operators offer differentiated products, with product i designating firm $i = 1, 2$'s product. In each location x, the single-product monopoly demand (for product 1) is $D_s(p_1)$, while the multi-product demand is $D_1(p_1, p_2)$ for product 1 and $D_2(p_2, p_1)$ for product 2. We normalize the firms' (constant) marginal cost of production to 0.

We adopt the linear demand model of Dixit (1979) and Singh and Vives (1984). The utility of the representative consumer is given by

$$U(q_1, q_2, m) = \alpha (q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2 + m,$$

where m is the numeraire good and $\gamma \in [0, 1)$ represents the degree of substitutability between products 1 and 2. The products are unrelated if $\gamma = 0$ and become perfect substitutes when $\gamma \to 1$. In the paper we will assume that $\gamma \leq 0.73$ to ensure the existence of a pure-strategy equilibrium.

If both goods 1 and 2 are available to the consumer, utility maximization yields the following multi-product demands for firms 1 and 2 (provided that quantities are positive),

$$D_1(p_1, p_2) = \frac{\alpha - p_1 - \gamma(\alpha - p_2)}{1 - \gamma^2} \text{ and}$$

$$D_2(p_1, p_2) = \frac{\alpha - p_2 - \gamma(\alpha - p_1)}{1 - \gamma^2}.$$

If only good 1 is available, the single-product demand for this good is

$$D_s(p_1) = \alpha - p_1.$$

For future use, we define the single-product monopoly price and the multi-product duopoly price as

$$p^m = \frac{\alpha}{2} \text{ and } p^d = \alpha\frac{1-\gamma}{2-\gamma},$$

respectively.

As Motta and Tarantino (2017), we study a simultaneous coverage-price game, where firms decide simultaneously on a coverage z_i for the technology and on a price p_i charged uniformly in all covered locations, with $i = 1, 2$. Note that this is equivalent to a situation where firms first decide on coverage and then on prices, but where coverage levels are not publicly observable when firms set their prices. In Appendix, we also present a sequential game, where firms first decide on coverage, then observe the coverage of their rival and set prices.

In the absence of merger, firm 1 and firm 2 are single-product firms. In the case of a merger, the merged entity offers the two products, 1 and 2, with potentially different coverage.

3. Analysis

We first determine the equilibrium of the coverage-price game without merger, and then with the merger. We finally compare the two equilibria to analyze the impact of a merger to monopoly on prices, coverage, and social welfare.

3.1. Without merger

Without a merger, firms 1 and 2 compete in coverage and prices.[7] Assuming that $z_1 \geq z_2$, firm 2 is competing on all its covered territory and faces the demand $z_2 D_2(p_2, p_1)$ over all locations, while firm 1 faces competition only on part of its territory, as it is the sole seller on all locations between z_2 and z_1, and faces the demand $z_2 D_1(p_1, p_2) + (z_1 - z_2) D_s(p_1)$. Firms' profits are then given by

$$\Pi_1 = z_2 p_1 D_1(p_1, p_2) + (z_1 - z_2) p_1 D_s(p_1) - C(z_1), \tag{1}$$

for firm 1, and

$$\Pi_2 = z_2 p_2 D_2(p_1, p_2) - C(z_2) \tag{2}$$

for firm 2.

Clearly, the pricing decision of firm 2 is the same as in the standard multi-product duopoly game, leading to the best-reply

$$p_2 = BR(p_1) = \frac{\alpha + \gamma(p_1 - \alpha)}{2}. \tag{3}$$

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[4] See Gilbert (2006) or Shapiro (2012) for a general discussion of the impact of mergers on innovative investment. In two recent contributions, Federico et al. (2017, forthcoming) argue that internalization by the merged firm of cannibalization of sales leads to a reduction of demand-enhancing efforts. Denicolò and Polo (2017) show however that their conclusion holds only if the R&D technology exhibits sufficient decreasing returns to scale.

[5] Davidson and Ferrett (2007) and Motta and Tarantino (2017) argue that sufficient synergies may stimulate post-merger investment.

[6] In any equilibrium, it is optimal for both firms to start their deployment at $x = 0$, as all locations are equivalent except for the investment cost. We thus focus on equilibria with coverage on a single interval.

[7] For an interior solution, we assume that c increases fast enough so that $c(0) < \pi^d = \alpha^2(1-\gamma)/((2-\gamma)^2(1+\gamma))$ and $c(\bar{z}) > \pi^m = \alpha^2/4$.

Firm 1 sets a uniform price for all its covered territory, balancing the revenue from the single-product part of it, where it faces no competition, and the revenue from the multi-product part, where it competes with firm 2. We find that:

$$p_1 = BR_1(p_2, z_1, z_2) = \frac{\alpha + \theta\gamma(p_2 - \alpha)}{2}, \tag{4}$$

where

$$\theta \equiv \frac{z_2}{(1-\gamma^2)z_1 + \gamma^2 z_2} \in [0, 1).$$

Thus, the larger the coverage by firm 2 (i.e., the closer it is to the coverage by firm 1), the closer is p_1 to the duopoly best-reply price.

Considering the choice of coverage by the two firms, we first notice that (see the Appendix), because expanding coverage yields higher returns for locations above z_2 (single-product) than below z_2 (multi-product), there is no equilibrium with symmetric coverage.

With a smaller coverage, firm 2 trades off the investment cost with the revenue of additional coverage in the multi-product zone. The equilibrium coverage for firm 2 is then the solution of

$$c(z_2) = p_2 D_2(p_1, p_2). \tag{5}$$

Firm 1, however, takes into account the difference in revenue between the single-product and multi-product zones. As the multi-product zone is infra-marginal, the marginal return on coverage is given by its profit in single-product locations, which yields:

$$c(z_1) = p_1 D_s(p_1). \tag{6}$$

We then obtain:

Proposition 1. *Under separation and sufficient product differentiation, a unique pure strategy equilibrium exists with* $z_1^S > z_2^S$ *and* $p^d < p_2^S < p_1^S < p^m$.

The firm with the largest coverage trades off between charging a high price to exploit its market power in single-product locations, and charging a low price to compete with its rival in multi-product locations. It thus sets a price in between its multi-product best-reply and the single-product monopoly price p^m, which is higher than the price set by the firm with a lower coverage. By strategic complementarity, both firms then set a price that is higher than the multi-product duopoly price p^d. Finally, coverage levels are:

$$c\left(z_1^S\right) = p_1^S\left(\alpha - p_1^S\right) = \frac{\alpha^2(2+\theta\gamma)\left[2-\theta\gamma(1+\gamma)\right]}{\left(4-\theta\gamma^2\right)^2} \tag{7}$$

and

$$c\left(z_2^S\right) = p_2^S D_2(p_1^S, p_2^S) = \left(\alpha\frac{2-\gamma-\theta\gamma^2}{4-\theta\gamma^2}\right)^2 \frac{1}{1-\gamma^2}, \tag{8}$$

with $\theta = \theta\left(z_1^S, z_2^S\right)$.

3.2. Merger-to-monopoly

If firms 1 and 2 merge, the merged entity has two products, 1 and 2, for which it can decide on different coverage levels. Assume that product 1 has a larger coverage, that is, $z_1 \geq z_2$. Given coverage z_1 and z_2 and prices p_1 and p_2 for products 1 and 2, respectively, the firm's profits are:

$$\Pi = (z_1 - z_2)p_1 D_s(p_1) + z_2\left[p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2)\right] - C(z_1) - C(z_2).$$

We find that the profit-maximizing prices are $p_1^M = p_2^M = p^m = \alpha/2$. The equilibrium coverage levels for products 1 and 2 are then given by

$$c\left(z_1^M\right) = \pi^m = \frac{\alpha^2}{4},\ c\left(z_2^M\right) = \max\left\{\left(\frac{1-\gamma}{1+\gamma}\right)\pi^m, c(0)\right\}.$$

The following proposition summarizes this analysis.

Proposition 2. *The merged firm sets* $p_1^M = p_2^M = p^m$*, and* $0 \leq z_2^M < z_1^M = c^{-1}(\pi^m)$.

As product substitutability decreases, the firm expands the range where it offers both products, while still offering only one product in the more costly locations.

3.3. Impact of merger on prices, coverage, and social welfare

We can now compare the equilibrium with and without the merger, in terms of prices and coverage.

Proposition 3. *A merger to monopoly raises prices and total coverage, and reduces the coverage of the multi-product zone.*

We have assumed that firms set their prices and coverage levels simultaneously. If coverage levels are observable before firms set prices, a new strategic effect arises. Firms account for the effect of coverage on prices, which leads the large firm to cover more and the small firm to cover less than when coverage is not observable. However, in this case too, we obtain that the merger raises total coverage.[8]

Proposition 3 shows that a merger to monopoly has a priori an ambiguous effect on social welfare. On the one hand, it leads to higher prices as expected. On the other, it raises the incentives to invest in total coverage. The multi-product coverage is however lower with the merger, and therefore there are locations where consumers lose the benefit from a larger variety of products.

Let us define welfare in a location x, denoted by w, as the sum of firms' profits (π_i) and consumer surplus (cs) in the location.

Without the merger, total equilibrium welfare is

$$W^S = z_1^S w^s(p_1^S) - C(z_1^S) + z_2^S \underbrace{\left(w^d(p_1^S, p_2^S) - w^s(p_1^S)\right)}_{>0} - C(z_2^S),$$

whereas if the firms merge, total welfare is

$$W^M = z_1^M w^s(p^m) - C(z_1^M) + z_2^M \underbrace{\left(w^d(p^m, p^m) - w^s(p^m)\right)}_{>0} - C(z_2^M).$$

The impact of the merger on total welfare is thus given by the difference:

$$\begin{aligned} W^M - W^S = {} & (z_1^M - z_1^S)w^s(p^m) - \left(C(z_1^M) - C(z_1^S)\right) \\ & + (z_2^M - z_2^S)\left(w^d(p^m, p^m) - w^s(p^m)\right) \\ & - \left(C(z_2^M) - C(z_2^S)\right) - \left(z_1^S - z_2^S\right)\left(w^s(p_1^S) - w^s(p^m)\right) \\ & - z_2^S\left(w^d(p_1^S, p_2^S) - w^d(p^m, p^m)\right). \end{aligned}$$

The first term corresponds to the positive[9] welfare gain from the total coverage expansion allowed by the merger, evaluated at constant prices. The second term corresponds to the effect of the reduction of the multi-product zone (again evaluated at constant prices). This term may be positive or negative depending on whether the incremental social value of the second product is smaller or larger than the incremental profit from it. Indeed, due to a standard business stealing effect (see Tirole, 1988), there may be excessive coverage by firm 2 in the duopoly game.[10]

Finally, the last line subsumes the traditional negative effect of the merger on prices in both regions, holding coverage constant.

8 See the Appendix.

9 It is positive because $w^s(p^m) > c\left(z_1^M\right) = \pi^m \geq c(z)$ for $z \leq z_1^M$.

10 While $w^d(p^m, p^m) - w^s(p^m) > c(z_2^M)$, it may be the case that $w^d(p^m, p^m) - w^s(p^m) < c(z_2^S)$, so this term cannot be signed.

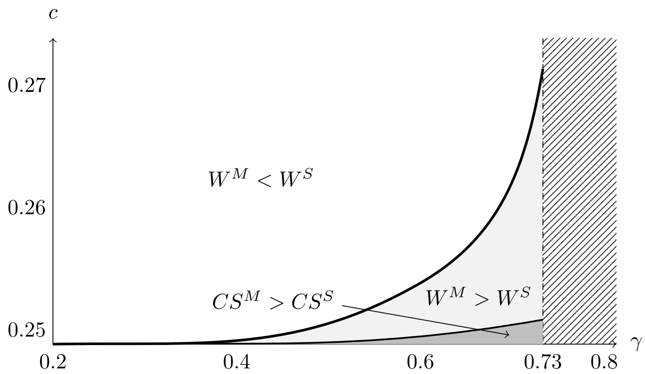

Fig. 1. Impact of merger on total welfare and consumer surplus ($\alpha = \beta = 1$).

To summarize, the merger expands total coverage, but has a negative impact on prices and variety. It has a positive effect on welfare if the expansion of total coverage is large enough and/or variety is excessive. We provide below a specific example where the merger leads to higher welfare.[11]

Proposition 4. *Suppose that* $C(z) = c \log\left[(\beta + e^z)/(1+\beta)\right]$ *and* $\gamma < 0.73$*. Then, a merger raises total welfare if* c *is not too high and* β *is not too small.*

Fig. 1 shows the area where a merger raises welfare (i.e., $W^M > W^S$) in the $\{\gamma, c\}$ space, for $\alpha = \beta = 1$ and our specific cost function.[12] This area corresponds to cases where, due to the low degree of differentiation: (i) competition between firms decreases total coverage substantially; (ii) the business stealing effect is strong. For consumers, we find that the set of parameter values such that consumer surplus increases with the merger is smaller but also non empty.

4. Conclusion

Our paper illustrates one channel through which a merger may have a positive effect on innovative investment: by raising equilibrium margins, a merger raises incentives to invest in coverage. In a merger case where investment in coverage is at stake, this positive effect on the incentives to invest has to be balanced with other countervailing effects on investment (Motta and Tarantino, 2017).

Our analysis readily extends to the case where the new technology replaces an old one, if innovation is drastic.[13] Due to a replacement effect (Arrow, 1962), the merger fosters the deployment of the new technology if the merger raises more the profits derived from the last location covered with the new technology than the profits obtained with the old technology, for instance because firms face competition in the old technology. The case where both the old and new technologies compete would require further analysis.

With more than two firms, our model can be viewed as describing the initial impetus of the merger, i.e., the initial incentives of merging firms for fixed behavior of competitors. Whether the reaction of competitors mitigates or exacerbates the effect of the merger on coverage is an open question that would call for a richer model.

11 We ran simulations with alternative cost functions. Welfare was systematically lower with the merger for the following cost functions: (i) $c(z) = cz^\beta$; and (ii) $c(z) = c\ \log(1+\beta z)$. With the cost functions (iii) $c(z) = cz^\beta/(1+z^\beta)$, and (iv) $c(z) = \beta c(e^z - 1)/(1+\beta)(e^z+\beta)$, we found similar results than the one given in the proposition.

12 Since $\pi^m = \alpha^2/4 = 1/4$, our assumption $c(0) < \pi^m < c(\infty)$ implies that $c \in (1/4, 1/2)$.

13 This is because only one technology is used at each location, so that the prices of one technology are not affected by the other technology.

Appendix

Lemma 1. *There is no equilibrium with symmetric coverage.*

Proof. With symmetric coverage $z_1 = z_2 = z$, we have $p_1 = p_2 = p^d = \alpha(1-\gamma)/(2-\gamma)$ and

$$p^d D_1\left(p^d, p^d\right) < p^d D_s\left(p^d\right),$$

implying that the right derivative of Π_1 in z_1 is strictly larger than its left derivative. As a consequence, if an equilibrium exists, it must be such that $z_1 > z_2$. ■

Proof of Proposition 1. Solving the first-order conditions (3) and (4) for the equilibrium prices $P_1(\theta)$ and $P_2(\theta)$, we find that for given coverage z_1 and z_2:

$$P_1(\theta) = \alpha\left(\frac{2-\theta\gamma-\theta\gamma^2}{4-\theta\gamma^2}\right) \text{ and}$$
$$P_2(\theta) = \alpha\left(\frac{2-\gamma-\theta\gamma^2}{4-\theta\gamma^2}\right). \tag{9}$$

Plugging the equilibrium prices into the demand functions, we obtain:

$$D_1(P_1, P_2) = \frac{\alpha\left(2-2\gamma-\gamma^2+\theta\gamma\right)}{\left(1-\gamma^2\right)\left(4-\theta\gamma^2\right)} \text{ and}$$
$$D_2(P_1, P_2) = \frac{\alpha\left(2-\gamma-\theta\gamma^2\right)}{\left(1-\gamma^2\right)\left(4-\theta\gamma^2\right)}. \tag{10}$$

We have $D_2(P_1, P_2) \geq 0$ for all $\theta \in [0, 1]$ and $\gamma \in [0, 1)$, while $D_1(P_1, P_2) \geq 0$ for all $\theta \in [0, 1]$ requires that $2-2\gamma-\gamma^2 \geq 0$, which is true since $\gamma \leq 0.73$ under our assumptions. For higher values of γ, that is, if the products are sufficiently homogeneous, firm 1 has an incentive to retreat on its single-product area by setting a high (monopoly) price. In this case, an equilibrium in pure strategy may fail to exist (see Valletti et al. (2002) and Gautier and Wauthy (2010) for details on this case).

The equilibrium prices $P_1(\theta)$ and $P_2(\theta)$ satisfy the following properties: (i) $P_i(\theta)$ is decreasing; (ii) $P_1(0) = p^m$; (iii) $P_2(0) = BR(p^m) < p^m$; and (iv) $P_1(1) = P_2(1) = p^d$. Moreover, the relative price of product 1 decreases with the relative coverage of product 1, i.e., with θ.

The first-order conditions for the coverage levels, (5) and (6), can then be rewritten as (we discuss the second-order conditions below):

$$c(z_1) = P_1(\theta) D_s(P_1(\theta)),$$
$$c(z_2) = P_2(\theta) D_2(P_1(\theta), P_2(\theta)).$$

These two equations define positive coverage levels $Z_1(\theta)$ and $Z_2(\theta)$, for $\theta \in [0, 1]$, both decreasing in θ. Since $Z_1(\theta) > Z_2(\theta)$, $Z_2(0) > 0$ and $Z_2(1) > 0$, we have

$$\frac{Z_2(0)}{(1-\gamma^2)Z_1(0)+\gamma^2 Z_2(0)} > 0 \text{ and}$$
$$\frac{Z_2(1)}{(1-\gamma^2)Z_1(1)+\gamma^2 Z_2(1)} < 1.$$

Hence, there exists $\theta \in (0, 1)$ such that

$$\frac{Z_2(\theta)}{(1-\gamma^2)Z_1(\theta)+\gamma^2 Z_2(\theta)} = \theta.$$

We thus have shown the existence of a solution to the system of first-order conditions, (3)–(6).

To prove that this is an equilibrium, we now exhibit conditions for the profit function of each firm to be quasi-concave.

Consider first firm 2. Its profit function, which is given by (2), is concave in price. Thus, its profit can be written as $\Pi_2(z_2) = z_2 \max_{p_2} p_2 D_2(p_1, p_2) - C(z_2)$, with a second derivative $\Pi_2''(z_2) = -c'(z_2) < 0$. Hence, firm 2's profit function is quasi-concave.

Consider now firm 1's profit, which can be written as

$$\Pi_1(z_1) = \max_{p_1} \{z_2 p_1 D_1(p_1, p_2) + (z_1 - z_2) p_1 D_s(p_1)\} - C(z_1).$$

We have

$$\Pi_1'(z_1) = p_1 D_s(p_1) - c(z_1),$$

with $p_1 = BR_1(p_2, z_1, z_2)$. The second derivative can then be written as:

$$\begin{aligned}\Pi_1''(z_1) &= \left(p_1 D_s'(p_1) + D_s(p_1)\right) \frac{\partial BR_1(p_2, z_1, z_2)}{\partial z_1} - c'(z_1) \\ &= (\alpha - 2p_1) \frac{\gamma}{2} (\alpha - p_2) \frac{(1-\gamma^2) z_2}{\left[z_1(1-\gamma^2) + z_2\gamma^2\right]^2} - c'(z_1).\end{aligned}$$

We look for a sufficient condition for the second derivative to be negative, therefore we look for an upper bound for $\Pi_1''(z_1)$. Since $p_i \geq p^d$, we have $(\alpha - 2p_1)(\alpha - p_2) \leq \alpha^2\gamma/(2-\gamma)^2$. Furthermore, since $z_1 \geq z_2$, we have

$$\frac{z_2}{\left[z_1(1-\gamma^2) + z_2\gamma^2\right]^2} \leq \frac{1}{z_2}.$$

Therefore, a sufficient condition for $\Pi_1''(z_1) \leq 0$ is that

$$\frac{1}{2} \frac{\alpha^2\gamma^2(1-\gamma^2)}{(2-\gamma)^2 z_2} \leq c'(z_1).$$

Finally, we have $z_1 > z_2 \geq c^{-1}\left(p^d D_2(p^d, p^d)\right) = c^{-1}(\pi^d)$, where $\pi^d \equiv \alpha^2(1-\gamma)/((2-\gamma)^2(1+\gamma))$ represents the multi-product duopoly profit. Therefore, we have $\Pi_1''(z_1) \leq 0$ if

$$\frac{1}{2} \frac{\alpha^2\gamma^2(1-\gamma^2)}{(2-\gamma)^2 c^{-1}(\pi^d)} \leq c'\left(c^{-1}(\pi^d)\right).$$

This condition can be written as

$$\frac{1}{2}\gamma^2(1+\gamma)^2 \leq \frac{c'\left(c^{-1}(\pi^d)\right) c^{-1}(\pi^d)}{\pi^d},$$

which holds for $\gamma \leq 0.73$ if at $x = c^{-1}(\pi^d)$ we have:

$$0.79746 \leq \frac{c'(x)\,x}{c(x)}.$$

Finally, since in equilibrium $\theta \in (0,1)$, then $p^d < P_2(\theta) < P_1(\theta) < p^m$. Furthermore, $\theta > 0$ implies that $z_1^S > z_2^S$. ■

Proof of Proposition 3. Taking the firm's profits gross of investment costs and dividing by $z_1 - z_2$, the firm sets its prices p_1 and p_2 to maximize

$$p_1(\alpha - p_1) + \frac{z_2}{z_1 - z_2}\left\{p_1\left(\frac{\alpha - p_1 - \gamma(\alpha - p_2)}{1-\gamma^2}\right) + p_2\left(\frac{\alpha - p_2 - \gamma(\alpha - p_1)}{1-\gamma^2}\right)\right\}.$$

From Propositions 1 and 2, we have $p_i^S < p^m = p_i^M$. Furthermore, since the right-hand side of (7) is decreasing in θ and $\theta\left(z_1^S, z_2^S\right) > 0$, we have $c\left(z_1^S\right) < c\left(z_1^M\right) = \pi^m$. Therefore, total coverage increases with the merger. Finally, note that the right-hand side of (8) is decreasing in θ, hence, it is minimum at $\theta = 1$. We find that

$$\left(\alpha\frac{2-\gamma-\gamma^2}{4-\gamma^2}\right)^2 \frac{1}{1-\gamma^2} \geq \left(\frac{1-\gamma}{1+\gamma}\right)\pi^m,$$

and hence $z_2^S \geq z_2^M$. ■

Sequential coverage-price game. Assume that firms first decide on coverage, and then observe the coverage levels and set prices. The equilibrium prices at the second stage are given by the same expressions than in the baseline model, given the coverage levels set at the first stage (see the expressions of prices in (9)). At the first stage, firms decide on coverage, anticipating the prices set in the second stage. Assume that $z_1 \geq z_2$. Using the envelope theorem, the equilibrium coverage levels are given by

$$c(z_1^S) = p_1^S D_s(p_1^S) + p_1^S z_2^S \frac{\partial D_1(p_1^S, p_2^S)}{\partial p_2}\frac{dp_2}{dz_1}, \tag{11}$$

and

$$c(z_2^S) = p_2^S D_2(p_1^S, p_2^S) + p_2^S z_2^S \frac{\partial D_2(p_1^S, p_2^S)}{\partial p_1}\frac{dp_1}{dz_2}. \tag{12}$$

We find that

$$\begin{aligned}&\frac{\partial D_1(p_1^S, p_2^S)}{\partial p_2} = \frac{\gamma}{1-\gamma^2} \quad \text{and} \\ &\frac{dp_2}{dz_1} = \frac{dp_2}{d\theta}\frac{d\theta}{dz_1} = \frac{\alpha\gamma^3(2+\gamma)\theta^2}{(4-\gamma^2\theta)^2},\end{aligned} \tag{13}$$

and therefore, the second term on the RHS of (11), which represents a strategic effect of coverage investment on the rival's price, is positive. Intuitively, by increasing its coverage, firm 1 softens price competition, which raises the marginal return on coverage.

Using (13), we can rewrite (11) as

$$\begin{aligned}c(z_1^S) = \alpha^2&\left(1 - \frac{2+\theta^S\gamma}{4-\theta^S\gamma^2}\right) \\ &\times\left(\frac{2+\theta^S\gamma}{4-\theta^S\gamma^2} + \left(\theta^S\right)^2\frac{\gamma^3(\gamma+2)}{\left(4-\theta^S\gamma^2\right)^2}\right).\end{aligned} \tag{14}$$

The comparison of $c\left(z_1^S\right)$ and $c\left(z_1^M\right) = \alpha^2/4$ shows that we have $z_1^M > z_1^S$ if and only if $8 - 4\gamma + \gamma^2(2+3\gamma)\theta > 0$, which is true for all $\gamma \in [0,1]$ and $\theta \in [0,1)$. ■

Proof of Proposition 4. In the case of a merger, we have prices $p_1^M = p_2^M = \alpha/2$ and quantities

$$q_1^{sM} = \frac{\alpha}{2} \text{ and } q_1^{dM} = q_2^{dM} = \frac{\alpha}{2}\frac{1}{1+\gamma},$$

leading to a point-wise welfare $w^s(p^m) = 3\alpha^2/8$ for the single-product zone and $w^d(p^m, p^m) = 3\alpha^2/[4(1+\gamma)]$ for the multi-product zone. The single-product and multi-product coverage are then given by

$$\begin{aligned}z_1^M &= c^{-1}\left(\frac{\alpha^2}{4}\right), \\ z_2^M &= c^{-1}\left(\frac{\alpha^2}{4}\frac{1-\gamma}{1+\gamma}\right) \text{ if } \frac{c}{1+\beta} < \frac{\alpha^2}{4}\frac{1-\gamma}{1+\gamma} \text{ and} \\ z_2^M &= 0 \text{ otherwise,}\end{aligned}$$

where $c^{-1}(y) = \log(\beta y/(c-y))$.

Consider now the case of separation. Equilibrium prices and quantities in the multi-product zone are given by Eqs. (9) and (10), where θ is endogenous. The quantity in the single-product zone is

$$q_1^s = \alpha - p_1^S = \alpha\left(\frac{2+\theta\gamma}{4-\theta\gamma^2}\right).$$

Finally, the single-product and multi-product coverage are given by

$$c\left(z_1^S\right) = \frac{\alpha^2\left(2+\theta\gamma\right)\left[2-\theta\gamma\left(1+\gamma\right)\right]}{\left(4-\theta\gamma^2\right)^2}$$

and

$$c\left(z_2^S\right) = \alpha^2\frac{\left(2-\gamma-\theta\gamma^2\right)^2}{\left(4-\theta\gamma^2\right)^2}\frac{1}{1-\gamma^2},$$

respectively, with $c(z) = c/(1+\beta e^{-z})$.

From the definition of θ, we have

$$\frac{z_2^S}{z_1^S} = \frac{\theta\left(1-\gamma^2\right)}{1-\theta\gamma^2}.$$

Combining these three equations leads to the following condition for θ:

$$\frac{c^{-1}\left(\alpha^2\frac{\left(2-\gamma-\theta\gamma^2\right)^2}{\left(4-\theta\gamma^2\right)^2}\frac{1}{1-\gamma^2}\right)}{c^{-1}\left(\frac{\alpha^2(2+\theta\gamma)[2-\theta\gamma(1+\gamma)]}{\left(4-\theta\gamma^2\right)^2}\right)} = \frac{\theta\left(1-\gamma^2\right)}{1-\theta\gamma^2}.$$

This allows to compute numerically all equilibrium quantities. ■

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